HIGH SCHOOL PHYSICAL SCIENCE

PART 2

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HIGH SCHOOL

PHYSICAL SCIENCE

PART II.

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CONTENTS.

PHYSICAL SCIENCE

PART II.

CHAPTER I.

VELOCITY.

The subjects of velocity and acceleration are treated experimentally in Chapter iii. of Part I. We shall now give general statements of the principles there discussed and a number of exercises containing problems in illustration of them.

1. Position.

Position is **relative**, not **absolute.** The position of a point P is determined when its **distance** and its **direction** from some other point, taken as a point of reference, are known.

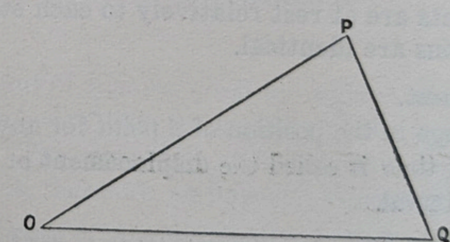

FIG. 1.

The line OP (Fig. 1) represents by its length and its direction the position of the point P with reference to the point O. Similarly, QP represents the position of the point P with reference to the point Q. That is, P has one position with reference to O, and another with reference to Q. In the same way, Q has one position, represented by the line OQ, with reference to O, and another, represented by the line PQ, with reference to P.

2. Motion.

A point is said to be in motion when its position is being changed continuously.

Motion, like position, is **relative.** A point is moving relatively to any point of reference when its position with respect to that point is changing continuously.

If the position of a point P with respect to a point of reference remains unchanged for a given time, P is said to be at rest with respect to this point of reference during that time; but, during the same time, the point P may be in motion with respect to another point of reference. A seat in a railway carriage in motion is at rest with respect to another seat in the same carriage, but in motion with respect to any object which has not the same motion as the carriage.

Two points are at rest relatively to each other when their motions are identical.

3. Displacement.

The change in the position of a point for any specified interval of time is called the **displacement** of the point for that interval.

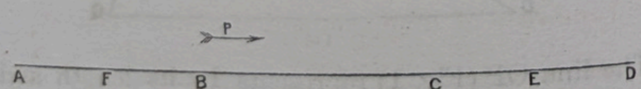

Fig. 2.

If the point P is in motion relatively to the point A in the line AD (Fig. 2), and if at any instant it is at B, and

at a subsequent instant at C, BC is the displacement of the point for the interval of time between these instants.

A displacement is determined when its magnitude and its direction are known.

The magnitude of the displacement is measured by the number of units of length contained in the line joining the two positions of the point.

If the point P has successive displacements BC, CD (Fig. 2) in the same direction, the total displacement equals

$$BC + CD,$$

the sum of these displacements; but if the point has displacement, BC, CD, DE, EF (Fig. 2), some in one direction and some in the opposite direction, and the displacements in opposite directions are given opposite signs, the total displacement in the positive direction equals

$$BC + CD - DE - EF,$$

the algebraic sum of the displacements.

4. Velocity.

The amount of time required for a given displacement of a point depends upon the rapidity of the movement of the point. The time-rate at which the displacement takes place is, therefore, an important quantity.

The time-rate of displacement is called velocity.

5. Uniform Velocity.

A point is said to be moving with a **uniform velocity** when it has equal displacements in equal intervals of time, however short these intervals may be.

6. Measure of Velocity.

A velocity, like a displacement, is determined when its magnitude and its direction are known. The magnitude of a velocity is measured by the number of times it contains some definite velocity, assumed as a unit. **The unit velocity is the velocity of a point which, moving uniformly, has a unit displacement in a unit of time.** The unit is a derived unit, because it is determined by the unit of length and the unit of time. For example, when the unit of length is the centimetre, and the unit of time the second, the unit of velocity is the centimetre per second.

With this system of units the measure of the velocity of a point moving uniformly is 1, when it has a displacement of 1 cm. in 1 second; 2, when it has a displacement of 2 cm. in 1 second; 3, when it has a displacement of 3 cm. in one second, and so on. If its displacement is 3 cm. in 2 seconds, in one second it is $\frac{3}{2}$ cm., and the measure of the velocity of the point is $\frac{3}{2}$.

In general terms, the measure of the velocity of a point moving uniformly, equals the measure of its displacement in a unit of time.

Thus, if s is the number of units of length in the displacement, and t the number of units of time in the interval, the measure of the displacement in a unit of time is $\frac{s}{t}$. Then, if v is the measure of the velocity,

$$v = \frac{s}{t}$$

or $$s = t\,v.$$

That is, the measure of the velocity of a point moving

uniformly during a given interval of time, **is obtained by dividing the measure of the displacement during that interval by the measure of the interval,**

or

the measure of the displacement equals the measure of the velocity multiplied by the measure of the interval.

EXERCISE I.

1. Express in miles per hour a velocity of (1) 40 feet per second, (2) 100 yards per minute.

2. A point moves at the rate of 50 miles in $1\frac{1}{2}$ hours. What is its velocity in feet per second?

3. Find the ratio of velocities of (1) 60 miles per hour and 44 feet per second, (2) 5 miles per 6 minutes and 10 feet per $\frac{1}{4}$ second.

4. One body moves over 30 yards in 7 minutes, and another over 12 feet in 5 seconds. If their velocities are uniform, compare them.

5. How many times does the velocity 176 yards per hour contain the velocity 2 miles per minute?

6. A velocity of 20 miles per hour is v times a velocity of 30 feet per second. What is v?

7. How many times does a velocity of 120 metres per minute contain a velocity of 20 cm. per sec.?

8. What is the measure of a velocity of 400 cm. per second when the unit velocity is (1) a centimetre per second, (2) a centimetre per 2 seconds, (3) a centimetre per $\frac{1}{2}$ second, (4) 2 centimetres per second, (5) $\frac{1}{2}$ centimetre per second?

9. What is the measure of a velocity of 120 metres per minute when 20 centimetres per second is the unit of velocity?

10. When 3 metres per 7 seconds is the unit of velocity, what is the measure of a velocity of 126 cm. per 3 seconds?

11. What is the measure of a velocity of 40 miles per hour when the unit velocity is (1) a foot per minute, (2) a foot per second, (3) a foot per 2 minutes, (4) 2 feet per second?

12. A point has a uniform velocity of 8 feet per second. What is its displacement in 11 hours?

13. A point is moving with a uniform velocity of 20 cm. per second. What is its displacement in metres in 10 hours?

14. A point moves uniformly in a straight line at the rate of a feet per second. What is its displacement in miles in b hours?

15. A point is moving uniformly at the rate of c cm. in s seconds. How far does it go in h hours?

16. The velocity of a train is 15 miles per hour. Find (1) how many minutes it will take to go 50 yards, (2) how many seconds it will take to go 25 feet.

17. The velocity of a point is a feet per b seconds. How long does it take it to go c miles?

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7. Velocity at an Instant.

An instant of time has no duration and therefore, while a point may be said to have a velocity at any given instant, an interval of time is necessary to produce a finite displacement, however small. For example, a falling body may be said to have a velocity of 10 feet per second at the instant it comes in contact with the earth, but at that instant its displacement is nothing. What is meant is, that if the body were to continue to move for one second with the velocity which it has at the instant it strikes the ground, it would have a displacement of 10 feet.

8. Average Velocity.

When a point is moving with a variable velocity, its **average velocity** for any given instant of time equals the uniform velocity of another point which has an equal total displacement in the interval.

Hence the measure of the average velocity during a given interval is obtained by dividing the measure of the distance traversed during that interval by the measure of the interval.

Thus, if s is the number of units in the total displacement, t the number of units of time in the interval, and v the measure of the average velocity, $v=\frac{s}{t}$.

If the velocity is increasing or decreasing uniformly, the average velocity for any given interval is the velocity at the middle instant of the interval. This equals half the sum of the velocities at the initial and the final instants of the interval.

If u is the measure of the velocity at the initial, and v the measure of the velocity at the final instant,

$$\text{the average velocity}=\frac{u+v}{2}$$

and if t is the measure of the interval, and s the measure of the displacement,

$$s=\left(\frac{u+v}{2}\right)t.$$

9. Measure of Variable Velocity.

If a point is moving with a varying velocity its actual velocity at a given instant may be defined as the average velocity during an infinitely short interval containing that instant.

A variable velocity may, therefore, be approximately measured by determining the average velocity for an interval containing the instant. It is manifest that the accuracy of the determination will depend upon the length of the interval. The shorter the interval the more accurate is the result. For example, the speed of a trolley car at a certain instant cannot be accurately determined by taking the average velocity between two stopping places; but, if the space traversed by the car in a very short interval containing the instant is observed, and the average velocity for the instant calculated, the result will be approximately the velocity of the car at the instant.

EXERCISE II.

1. A point has displacements of 3 feet, 4 feet, 5 feet, and 6 feet in four consecutive seconds. What is its average velocity?

2. A point has displacements of 9 cm., 10 cm., 11 cm., and 12 cm. in four consecutive seconds. Find its average velocity (1) for the four seconds, (2) for the first three seconds, (3) for the last three seconds.

3. A point is displaced 5 cm., 3 cm., 1 cm., -1 cm., -3 cm. in five consecutive seconds. What is its average velocity (1) for the five seconds, (2) for the first 3 seconds, (3) for the last three seconds, (4) for the middle three seconds?

4. A point has displacements of 2 feet, 6 feet, 10 feet, 14 feet, and 18 feet in five consecutive seconds. Show that the average velocities for the five seconds, the middle second, and the three middle seconds, are all equal.

5. A point moving with a uniformly increasing velocity has a velocity of 10 feet per second at the beginning of a certain interval of 5 seconds, and a velocity of 20 feet per second at the end of the interval. Find (1) its average velocity, (2) its displacement, during the interval.

6. A point moving with a uniformly decreasing velocity has a velocity of 60 cm. per second at the beginning of a certain interval of 10 seconds, and a velocity of 10 cm. per second at the end of the interval. Find (1) the velocity at the middle instant of the interval, (2) the displacement during the interval.

7. A point has a velocity of 10 cm. per second at the beginning, and 12 cm. per second at the end of a certain second. If its velocity is increasing uniformly, find (1) its velocity at the end of three seconds more, (2) its average velocity for the four seconds, (3) its displacement for the four seconds.

8. In one hour the velocity of a point decreases uniformly from 200 feet per second to 100 feet per second. What is the velocity at the end of each quarter of an hour, and what is the total displacement during the hour?

9. At 9 A.M. a point is moving with a velocity of 40 cm. per second, and at 1 P.M. it has a velocity of 120 cm. per second. If it moves with a uniformly increasing velocity, what velocity will it have at 11.30 A.M.?

10. At 1 P.M. a point has a velocity of 60 feet per second, and in one minute its velocity increases to 65 feet per second. If the velocity continues to increase uniformly at the same rate, find (1) its velocity at 2.15 P.M., (2) how far it goes between 1.15 and 1.20 P.M., (3) how far it goes between 2.10 and 2.15 P.M.

11. A point is moving with a uniformly decreasing motion. At the beginning of a certain second its velocity is 20 feet per second, and at the end of the same second its velocity is 18 feet per second. When will it come to rest? How far will it go during the second before it comes to rest?

12. A point is moving at a given instant with a velocity of 8 feet per second. At the end of 5 seconds its velocity is 19 feet per second. When will it have a velocity of (1) 16.8 feet per second, (2) 30 feet per second?

13. A point, starting from rest, has its velocity increased uniformly 5 feet per second each second. Find (1) the velocity at the end of 10 seconds, (2) the displacement of the point during that time.

14. A point which has a velocity of 80 centimetres per second has its velocity uniformly decreased by 4 cm. per second each second. How long before it will come to rest, and how far will it move during the time?

15. The velocity of a point increases uniformly in 10 seconds from 150 cm. per second to 200 cm. per second. Find (1) the rate of increase in velocity, (2) the displacement during the 10 seconds.

16. A train moving with a velocity of 60 kilometres per hour is pulled up with a uniformly decreasing velocity in 30 seconds. What is the decrease in velocity during each second in centimetres per second?

CHAPTER II.

ACCELERATION.

1. Uniform Acceleration.

If the motion of a point is changing, the point is said to be accelerated positively or negatively, according as its velocity is increasing or diminishing.

Rate of change of velocity is called Acceleration.

The acceleration is **uniform** when equal changes of velocity take place in equal intervals of time, however short these intervals may be.

2. Measure of Uniform Acceleration.

Since acceleration is the rate of change of velocity, or the change of velocity in a unit of time, acceleration is measured by a unit of acceleration derived from the unit of velocity and the unit of time.

The unit acceleration is the acceleration of a point, the motion of which is such that its velocity is increased, or diminished, by the unit of velocity in each unit of time.

For example, if the centimetre is taken as the unit of length and the second as the unit of time, the **unit velocity is the centimetre per second,** and the **unit acceleration the centimetre per second per second.** An acceleration of 1 cm. per second per second is a change in velocity, during one second, of one centimetre per second; an acceleration of 2 cm. per second per second is a change in velocity, during one second, of two centimetres per second; and an acceleration of a cm. per second per second, a change in velocity, during one second, of a centimetres per second.

If v is the measure of the change in velocity of a point in an interval of time, the measure of which is t, $\frac{v}{t}$ is the measure of the change of velocity in a unit of time. Then, if a is the measure of the acceleration, or rate of change of velocity,

$$a = \frac{v}{t}$$

$$\text{or} \qquad v = a\,t.$$

That is, the **measure of the uniform acceleration** of a point during a given interval of time is **obtained by dividing the measure of the change of velocity during that interval by the measure of the interval,**

or

the measure of the change in velocity equals the measure of the acceleration multiplied by the measure of the interval.

EXERCISE III.

1. A point is moving with a uniform acceleration of 10 units of velocity per second. (1) What velocity will it acquire in a minute? (2) What will be the acceleration in units of velocity per minute?

2. A point is moving with a uniform acceleration of 10 feet per second per second. (1) What is the total change in velocity in a minute? (2) What is the measure of the acceleration in feet per second per minute?

3. What velocity will a body acquire in half an hour if the acceleration is (1) 10 centimetres per second per minute, (2) 10 centimetres per second per second?

4. A point is travelling with an acceleration of 12 feet per second per hour. Find (1) what will be its change in velocity in a minute, (2) the measure of its acceleration in feet per second per second?

5. A train acquires a velocity of 30 feet per second in one hour. If its velocity is uniformly accelerated, find (1) the velocity which it will acquire in one minute, (2) the measure of its acceleration in feet per second per second.

6. A point is travelling with an acceleration of 12 feet per second per hour. How long will it take it to acquire a velocity of 2 feet per second?

7. A train, moving with a uniform acceleration, acquires a velocity of 75 feet per second in a quarter of a minute. How long will it take it to acquire a velocity of 100 yards per minute?

8. A point, moving with a uniform acceleration, acquires a velocity of 60 feet per second in 10 minutes. What is the measure of its acceleration in (1) feet per second per minute, (2) yards per second per minute, (3) feet per second per second, (4) yards per second per second?

9. A point, travelling with a uniform acceleration, has its velocity increased 50 metres per second each minute. What is the measure of the acceleration in (1) metres per second per minute, (2) centimetres per second per minute, (3) metres per second per second, (4) centimetres per second per second?

10. A train, moving with a uniform acceleration, acquires an additional velocity of 60 feet per second each minute. Find (1) the measure of its acceleration in feet per second per second, (2) the measure of the velocity it acquires each minute in feet per minute, (3) the measure of the acceleration in feet per minute per minute, (4) the measure of the acceleration in feet per minute per second.

11. A point is moving with a uniform acceleration and acquires an additional velocity of 20 cm. per second each second. Find the measure of the acceleration in (1) centimetres per second per

minute, (2) centimetres per minute per minute, (3) metres per minute per minute, (4) metres per minute per second, (5) metres per second per second.

12. What is the measure of an acceleration of 30 feet per second per second when the units of displacement and of time are respectively (1) the foot and the second, (2) the foot and the minute?

13. The measure of the acceleration of a falling body is 32 when the foot is the unit space and the second the unit of velocity. What is the measure of this acceleration when the yard is the unit of space and the minute is unit of time?

14. The acceleration due to gravity of a body is 980 centimetres per second per second. What is the measure of this acceleration when the metre is the unit of displacement and the minute the unit of time?

15. Compare an acceleration of 120 feet per second per minute with an acceleration of 20 yards per minute per second.

16. How many times does an acceleration of 5 feet per second per second contain an acceleration of 18 feet per minute per minute?

17. What is the measure of an acceleration of 5 feet per second per second when 18 feet per minute per minute is the unit of acceleration?

18. Compare an acceleration of which the measure is 150 when the yard is the unit displacement and the minute the unit of time with an acceleration of which the measure is .25 when the foot is the unit of displacement and the second the unit of time.

19. What is the measure of an acceleration of 150 yards per minute per minute when an acceleration of .25 feet per second per second is the unit acceleration?

We have in the preceding exercises given a series of questions which illustrate the relations which exist among the quantities involved in problems relating to uniformly accelerated velocities. We shall now derive equations which state in a general way these relations.

Let a be the measure of the acceleration of a point moving with a uniform acceleration.

t, the measure of any interval of time in its motion.

s, the measure of the displacement for the interval.

u, the measure of the velocity of the point at the beginning of the interval.

v, the measure of the velocity of the point at the end of the interval.

Then,

$v-u=$ the change in velocity in t units of time,

$\frac{v-u}{t}=$ the change in velocity in one unit of time,

$=$ the rate of change in velocity,

$=a$,

or, $$\mathbf{v}=\mathbf{u}+a\,\mathbf{t} \quad . \; . \; . \; . \; . \quad \text{(i.)}$$

If the point starts from rest, $u=o$ and $v=a\,t$.

Therefore, if the acceleration remains constant,

$$v \propto t.$$

Again, since the velocity increases or decreases uniformly,

the average velocity $=$ the velocity at the middle instant of the interval,

$=$ half the sum of the initial and the final velocities,

$$=\frac{u+v}{2}$$

But the displacement = the average velocity × time (Art. 8, page 7). Therefore,

$$s = \left(\frac{u+v}{2}\right) t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii.)}.$$

Substituting the value for v given in (i.) for v in (ii.), we have,

$$s = \frac{u + u + a\,t}{2} \times t$$

or,

$$s = u\,t + \tfrac{1}{2}\,a\,t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iii.)}.$$

It should be carefully noted that when the initial velocity is in the opposite direction to the acceleration, that is when u and a are of opposite signs, the value of s given in this equation does not always indicate the whole distance travelled by the point during the interval of time, but simply the distance between its positions at the beginning and at the end of the interval.

If the point starts from rest, $u = o$, and

$$s = \tfrac{1}{2}\,a\,t^2.$$

Therefore, if the acceleration is constant,

$$s \propto t^2.$$

From (i.)

$$v = u + a\,t,$$

or,

$$t = \frac{v - u}{a}$$

From (ii.)

$$s = \left(\frac{v + u}{2}\right) t$$

Substituting the value for t given in (i.) for t in (ii.),

We have

$$s = \frac{v+u}{2} \times \frac{v-u}{a} = \frac{v^2 - u^2}{2\,a}$$

or

$$v^2 = u^2 + 2\,a\,s. \qquad \text{(iv.)}.$$

If the point starts from rest, $u = o$, and

$$v^2 = 2\,a\,s.$$

Therefore, if the acceleration is constant,

$$s \propto v^2.$$

It will be noticed that each of the equations,

$$v = u + a\,t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i.)}.$$

$$s = \left(\frac{u+v}{2}\right) t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii.)}.$$

$$s = u\,t + \tfrac{1}{2}\,a\,t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iii.)}.$$

$$v^2 = u^2 + 2\,a\,s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iv.)}.$$

gives the relation among four of the five quantities **u**, **v**, **t**, a, and **s**, and that, if any three of these are known, the values of the other two may be derived from the equations.

EXERCISE IV.

1. What is the initial velocity of a point which, moving with a uniform acceleration of 10 centimetres per second per second, acquires in 10 seconds a velocity of 200 centimetres per second?

2. A body, moving at a certain instant with a velocity of 30 miles per hour, is subject to a uniform acceleration in the opposite direction, and comes to rest in 11 seconds. What was the measure of its velocity, in feet per second, 5 seconds before it stopped?

3. Find the initial velocity of a point which moves with a uniform acceleration of 20 centimetres per second per second, and acquires a velocity of 15 centimetres per second in 10 seconds. Interpret the result.

4. The velocity of a point increases uniformly in 20 seconds from 100 centimetres per second to 200 centimetres per second. Find (1) the measure of the acceleration in centimetres per second per second, (2) the velocity three seconds after it was 150 centimetres per second, (3) when the body was at rest.

5. A point, which has an acceleration of 32 feet per second per second, is moving with a velocity of 10 feet per second. At the same place and at the same time another point, which has an acceleration of 16 feet per second per second, is moving in the same direction with a velocity of 170 feet per second. Find (1) when the two points will have equal velocities, (2) when the velocity of the second will be double that of the first.

6. A body, moving with a velocity of 5 centimetres per second, has a constant acceleration of 10 centimetres per second per second in the direction of its motion. Find (1) how far it will go in 10 seconds, (2) how long it will take it to go 10 centimetres.

7. A body starts with a velocity of 15 centimetres per second, and has a constant acceleration of 10 centimetres per second per second in the opposite direction. When and where will it come to rest?

8. A body, starting from rest, moves with a uniform acceleration of 20 feet per second per second. Find (1) how far the body goes in 4 seconds, (2) how far it goes in five seconds, (3) how far it goes in the fifth second.

9. A body starts with a velocity of 6 feet per second and has a uniform acceleration of 3 feet per second per second in the direction of its motion. At the end of four seconds the acceleration ceases. How far does the body move in 10 seconds from the beginning of its motion?

10. With what uniform acceleration does a point, starting from rest, describe 640 feet in 8 seconds?

11. A point, starting from rest and moving with a uniform acceleration, has a displacement of 66 feet in the 6th second. What is the measure of the acceleration in feet per second per second, and what is its displacement in the 7th second?

12. A body, moving with a uniform acceleration, has displacements in the 4th and the 6th seconds from starting of 38 feet and 54 feet respectively. Find (1) the acceleration, (2) the initial velocity, (3) the displacement in the 7th second.

13. A particle, moving with a uniform acceleration, has a displacement of 30 centimetres in the first 2 seconds of its motion. The acceleration then ceases, and the displacement for the next 2 seconds is 20 centimetres. Find (1) the initial velocity, (2) the acceleration.

14. A train, having a velocity of 20 feet per second, attains a velocity of 30 miles per hour in passing over 128 feet. If the train is moving with a uniform acceleration, what is its acceleration?

15. A trolley car, moving at the rate of 24 feet per second, is stopped with a uniformly decreasing motion in a space of 9 feet. What is the acceleration of the car?

16. A particle starts with a velocity of 23 feet per second, and its velocity is uniformly decreased at the rate of 8 feet per second per second. Find how long it will take it to describe a distance of 30 feet, and how much longer to come to rest.

17. The displacement of a point moving with a uniformly decreasing motion is 100 metres in 10 seconds, and 100 metres in the next 12 seconds. In what time will it be 100 metres further on?

18. What is the velocity of a particle which, starting from rest and moving with a uniform acceleration of 8 feet per second per second, has traversed 100 feet? Find also the time required for this displacement.

19. A point which has a negative uniform acceleration of 10 centimetres per second per second is passing a certain point, and at the end of 15 centimetres farther on it has a velocity of 10 centimetres per second. Find (1) its velocity at the point, (2) in how many seconds it will return to this point.